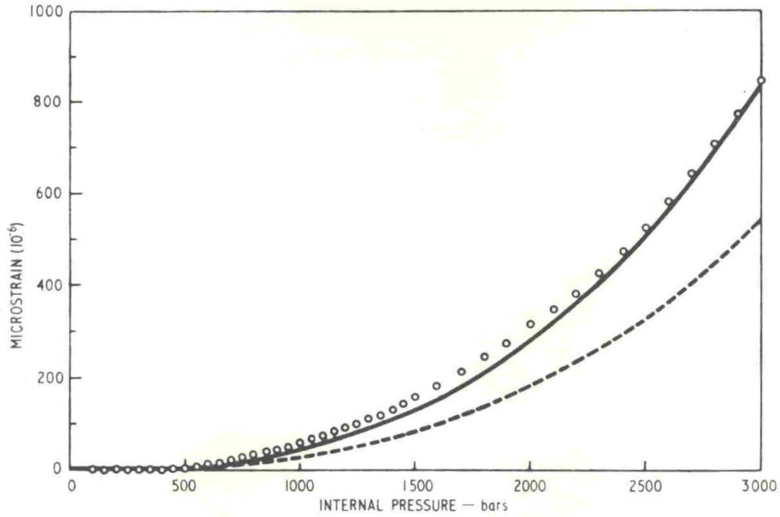

- - - - Calculation based on the Lévy–von Mises strain relation.
——— Calculation based on the new strain law proposed.
○ Experimental data.

Fig. 7. Comparison of the calculated external circumferential strains with those measured on raising the pressure in a cylinder with an r_o/r_n ratio of 2 made from steel D

an angle of 45° with the directions of the principal stresses, each of these strains depending solely on the shear stress in the corresponding plane. Application of this method to two individual cases shows that the stress distribution is highly dependent on the law of microplasticity of the material. The external circumferential strains derived by this method agree closely with those observed experimentally while the pressure builds up in the cylinder.

ACKNOWLEDGEMENTS

The authors wish to thank CRIF (Centre de Recherches Scientifiques et Techniques de l'Industrie des Fabrications Métalliques) and IRSIA (Institut pour l'Encouragement de la Recherche Scientifique dans l'Industrie et l'Agriculture) for supporting the cost of the present work.

APPENDIX 1

Calculation of the stress distribution in a thick-walled cylinder subjected to an internal pressure

A method is proposed for calculating the stress and strain distribution in a long, thick-walled, closed-end cylinder subjected to an internal pressure. For this calculation it will be assumed that the law of microplasticity of the material can be depicted by a relation of the type

$$\epsilon_p = f(\sigma) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In the case of steel, this relation can take the form of equation (2). For other materials, other relations can be used without calling for any modification of the calculation method. It will further be assumed that the cylinder has never been subjected to an internal pressure. This restriction is related to the fact that equation (2) holds only for a metal which has not been stressed previously.

Based on considerations of symmetry, it will also be assumed that the directions of the principal stresses are known. These principal stresses, σ_r, σ_t, σ_a, are parallel to the radius, the tangent, and the axis of the cylinder, respectively. Due to the symmetry of revolution, these three stresses do not vary with the angle, θ. Furthermore, in the case of a cylinder of sufficient length, σ_r, σ_t, and σ_a can be assumed to be independent of z. Thus, the only variable to be considered is the radius r. It will finally be assumed that a plane section perpendicular to the axis of the cylinder remains plane after deformation: in other words, the total elongation parallel to the axis (ϵ_a) does not depend on r.

Equations to describe the behaviour of material

In the most general case, the ϵ_r, ϵ_t, and ϵ_a strains, which are parallel to the radius, tangent, and axis, comprise an elastic component and a plastic component, though the latter can be nil:

$$\epsilon_t = \epsilon_{te} + \epsilon_{tp} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Two similar equations hold for ϵ_r and ϵ_a. The elastic strains are given by

$$\epsilon_{te} = \frac{1}{E}[\sigma_t - \nu(\sigma_a + \sigma_r)] \quad . \quad . \quad . \quad (8)$$

and similar equations for ϵ_{re} and ϵ_{ae}, where E is Young's modulus and ν is Poisson's ratio. The microplastic strains are given by three equations of the type

$$\epsilon_{tp} = \tfrac{1}{2}f(\sigma_t - \sigma_a) + \tfrac{1}{2}f(\sigma_t - \sigma_r) \quad . \quad . \quad (9)$$

Method of calculation

The cylinder is divided into n concentric zones of equal thicknesses, the thickness of each zone being small with respect to the wall thickness. Calculation is started from the outer surface of the cylinder and progresses inwards, the zones being numbered $1, 2, \ldots, i, \ldots, n$ in this direction. The principal stresses at the outer radius (r_{i-1}) of the zone, i, viz.

$$\sigma_{r_{i-1}}, \sigma_{t_{i-1}}, \sigma_{a_{i-1}} \quad . \quad . \quad . \quad (10)$$

are known. To calculate the three corresponding stresses at the inner diameter of the same zone, viz.

$$\sigma_{r_i}, \sigma_{t_i}, \sigma_{a_i} \quad . \quad . \quad . \quad . \quad (11)$$